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SOLVABILITY OF A SYSTEM OF GENERALIZED MIXED VARIATIONAL INEQUALITIES

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Abstract: The aim of this work is to study a system of generalized mixed variational inequality and its approximate solvability using the resolvent operator technique. The results presented in this work are more general and include many previously known results as special cases.

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1. Introduction and Preliminaries

Let H be a real Hilbert space whose inner product and norm are denoted by $\langle\cdot,\cdot\rangle$ and $\|\cdot\|$, respectively. Let $\varphi : H \to (-\infty,+\infty)$ be a proper convex lower semi-continuous function on H. Let $T, g : H \to H$ be nonlinear operators. In 1999, Noor [6] studied the problem of finding $x^* \in H$ such that

$$\langle Tx^*, g(y^*) - g(x^*)\rangle + \varphi(g(y^*)) - \varphi(g(x^*)) \geq 0\,, \quad \forall\, g(y^*) \in H\,. \tag{1}$$

The inequality of type (1) is called the general mixed variational inequality. A wide class of linear and nonlinear problems arising in pure and applied sciences can be studied via the general mixed variational inequalities (1).

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We consider the following system of generalized mixed variational inequality problem (SGMVI): find $x^*, y^*, z^* \in H$ such that

$$\begin{aligned} &\langle \rho T_1(y^*, x^*) + g(x^*) - g(y^*), x - g(x^*)\rangle + \varphi(x) - \varphi(g(x^*)) \geq 0, \\ &\langle \eta T_2(x^*, y^*) + g(y^*) - g(x^*), x - g(y^*)\rangle + \varphi(x) - \varphi(g(y^*)) \geq 0, \end{aligned} \tag{2}$$

for all $x \in H$, $\rho > 0, \eta > 0$, where $T_1, T_2 : H \times H \to H$, $g : H \to H$ are nonlinear operators.

If $T_1 = T_2 = T$ and $g = I$, then the problem (SGMVI) reduces to the following system of mixed variational inequalities considered by [3, 7] for finding $x^*, y^* \in H$ such that

$$\begin{aligned} &\langle \rho T(y^*, x^*) + x^* - y^*, x - x^*\rangle + \varphi(x) - \varphi(x^*) \geq 0\,, \\ &\langle \eta T(x^*, y^*) + y^* - x^*, x - y^*\rangle + \varphi(x) - \varphi(y^*) \geq 0\,, \end{aligned} \tag{3}$$

for all $x \in H$, $\rho > 0, \eta > 0$.

If K is closed convex set in H and $\varphi(x) = \delta_K(x)$, for all $x \in K$, where δ_K is the indicator function of K defined by

$$\delta_K(x) = \begin{cases} 0, & \text{if } x \in K\,; \\ +\infty, & \text{otherwise}\,, \end{cases}$$

then the problem (2) reduces to the following system of general variational inequality problem: Find $x^*, y^* \in K$ such that

$$\begin{aligned} &\langle \rho T_1(y^*, x^*) + g(x^*) - g(y^*), x - g(x^*)\rangle \geq 0\,, \\ &\langle \eta T_2(x^*, y^*) + g(y^*) - g(x^*), x - g(y^*)\rangle \geq 0\,, \end{aligned} \tag{4}$$

for all $x \in K$, $\rho > 0, \eta > 0$.

The problem (4) has been studied in [11].

If $T_1 = T_2 = T$ and $g = I$, then the problem (2) reduces to the following system of general variational inequality problem: find $x^*, y^* \in K$ such that

$$\begin{aligned} &\langle \rho T(y^*, x^*) + x^* - y^*, x - x^*\rangle \geq 0\,, \\ &\langle \eta T(x^*, y^*) + y^* - x^*, x - y^*\rangle \geq 0\,, \end{aligned} \tag{5}$$

for all $x \in K$, $\rho > 0, \eta > 0$. The problem (5) is studied by Verma [8, 9] and Chang et al. [2].

For a multivalued operator $T : H \to H$, we denote by

$$D(T) = \{u \in H : T(u) \neq \emptyset\}\ ,$$

the domain of T,

$$R(T) = \bigcup_{u \in H} T(u)\,,$$

the range of T,

$$\mathrm{Graph}(T) = \{(u, u^*) \in H \times H : u \in D(T) \text{ and } u^* \in T(u)\}\ ,$$

the graph of T.

Definition 1. T is called monotone if and only if for each $u \in D(T)$, $v \in D(T)$ and $u^* \in T(u)$, $v^* \in T(v)$, we have

$$\langle v^* - u^*, v - u \rangle \geq 0\,.$$

T is maximal monotone if it is monotone and its graph is not properly contained in the graph of any other monotone operator.

T^{-1} is the operator defined by

$$v \in T^{-1}(u) \Leftrightarrow u \in T(v)\,.$$

Definition 2. (see [1]) For a maximal monotone operator T, the resolvent operator associated with T, for any $\sigma > 0$, is defined as

$$J_T(u) = (I + \sigma T)^{-1}(u)\,, \ \ \forall\, u \in H\,.$$

It is known that a monotone operator is maximal if and only if its resolvent operator is defined everywhere. Furthermore, the resolvent operator is single-valued and nonexpansive, i.e., $\|J_T x - J_T y\| \leq \|x - y\|\,,\ \forall x, y \in H$. In particular, it is well known that the subdifferential $\partial\varphi$ of φ is a maximal monotone operator, see [4].

Lemma 3. (see [1]) *For a given $y, z \in H$ satisfies the inequality*

$$\langle u - z, x - u \rangle + \lambda\varphi(x) - \lambda\varphi(u) \geq 0\,, \ \ \forall x \in H$$

if and only if $u = J_\varphi(z)$, where $J_\varphi = (I + \lambda\,\partial\varphi)^{-1}$ is the resolvent operator and $\lambda > 0$ is a constant.

Using Lemma 3, we will establish the following important relation:

Lemma 4. *The variational inequality problem* (2) *is equivalent to finding* $x^*, y^* \in H$ *such that*

$$\begin{aligned} g(x^*) &= J_\varphi\left[g(y^*) - \rho T_1(y^*, x^*)\right], \\ g(y^*) &= J_\varphi\left[g(x^*) - \eta T_2(x^*, y^*)\right], \end{aligned} \tag{6}$$

where $\rho, \eta > 0$ *and* $J_\varphi = (I + \partial\varphi)^{-1}$.

Proof. Let $x^*, y^* \in H$ be a solution of (2). Then for all $x \in H$, we have

$$\langle \rho T_1(y^*, x^*) + g(x^*) - g(y^*), x - g(x^*)\rangle + \varphi(x) - \varphi(g(x^*)) \geq 0,$$
$$\langle \eta T_2(x^*, y^*) + g(y^*) - g(x^*), x - g(y^*)\rangle + \varphi(x) - \varphi(g(y^*)) \geq 0,$$

which can be written as

$$\begin{aligned} &\langle g(x^*) - \left[g(y^*) - \rho T_1(y^*, x^*)\right], x - g(x^*)\rangle + \varphi(x) - \varphi(g(x^*)) \geq 0, \\ &\langle g(y^*) - \left[g(x^*) - \eta T_2(x^*, y^*)\right], x - g(y^*)\rangle + \varphi(x) - \varphi(g(y^*)) \geq 0, \end{aligned} \tag{7}$$

using Lemma 3 for $\lambda = 1$, we can see that (7) is equivalent to

$$\begin{aligned} g(x^*) &= J_\varphi\left[g(y^*) - \rho T_1(y^*, x^*)\right], \\ g(y^*) &= J_\varphi\left[g(x^*) - \eta T_2(x^*, y^*)\right]. \end{aligned}$$

This completes the proof. □

Lemma 4 implies that the system of general mixed variational inequality problem (2) is equivalent to fixed point problem. This alternative equivalent formulation if very useful from numerical point of view. Using this fixed point formulation, we suggest and analyze the following iterative algorithm.

Algorithm 1. For arbitrary chosen points $x_0, y_0 \in H$, compute the sequences $\{x_n\}$, $\{y_n\}$ such that

$$\begin{aligned} g(x_{n+1}) &= (1 - \alpha_n) g(x_n) + \alpha_n J_\varphi\left[g(y_n) - \rho T_1(y_n, x_n)\right] \\ g(y_n) &= (1 - \beta_n) g(x_n) + \beta_n J_\varphi\left[g(x_n) - \eta T_2(x_n, y_n)\right], \end{aligned} \tag{8}$$

where $J_\varphi = (I + \partial\varphi)^{-1}$ is the resolvent operator, $\{\alpha_n\}$, $\{\beta_n\}$ are sequences in $[0, 1]$ and ρ, η are positive real numbers.

If $T_1, T_2 : H \to H$ be univariate mapping, then Algorithm 1 reduces to the following:

Algorithm 2. For arbitrary chosen points $x_0, y_0 \in H$, compute the sequences $\{x_n\}$, $\{y_n\}$ such that

$$\begin{aligned} g(x_{n+1}) &= (1-\alpha_n)g(x_n) + \alpha_n J_\varphi\left[g(y_n) - \rho T_1(y_n)\right] \\ g(y_n) &= (1-\beta_n)g(x_n) + \beta_n J_\varphi\left[g(x_n) - \eta T_2(x_n)\right], \end{aligned} \tag{9}$$

where $J_\varphi = (I + \partial\varphi)^{-1}$ is the resolvent operator, $\{\alpha_n\}$, $\{\beta_n\}$ are sequences in $[0,1]$ and ρ, η are positive real numbers.

If $\beta_n = 1$, then Algorithm 1 reduces to the following:

Algorithm 3. For arbitrary chosen points $x_0, y_0 \in H$, compute the sequences $\{x_n\}$, $\{y_n\}$ such that

$$\begin{aligned} g(x_{n+1}) &= (1-\alpha_n)g(x_n) + \alpha_n J_\varphi\left[g(y_n) - \rho T_1(y_n, x_n)\right] \\ g(y_n) &= J_\varphi\left[g(x_n) - \eta T_2(x_n, y_n)\right], \end{aligned} \tag{10}$$

where $J_\varphi = (I + \partial\varphi)^{-1}$ is the resolvent operator, $\{\alpha_n\}$ is a sequence in $[0,1]$ and ρ, η are positive real numbers.

If $T_1, T_2 : H \to H$ are univariate mapping and $\beta_n = 1$, then Algorithm 1, reduces to the following:

Algorithm 4. For arbitrary chosen points $x_0, y_0 \in H$, compute the sequences $\{x_n\}$, $\{y_n\}$ such that

$$\begin{aligned} g(x_{n+1}) &= (1-\alpha_n)g(x_n) + \alpha_n J_\varphi\left[g(y_n) - \rho T_1(y_n)\right] \\ g(y_n) &= J_\varphi\left[g(x_n) - \eta T_2(x_n)\right], \end{aligned} \tag{11}$$

where $J_\varphi = (I + \partial\varphi)^{-1}$ is the resolvent operator, $\{\alpha_n\}$ is a sequence in $[0,1]$ and ρ, η are positive real numbers.

2. Main Result

We now present the approximation solvability of the problem (2). For this purpose we first give some definitions:

Definition 5. An operator $T : H \times H \to H$ with respect to an arbitrary operator g is said to be :

(i) (g, ν) strongly monotone, if for each $x, x' \in H$, there exists a constant $\nu > 0$ such that

$$\langle T(x,y) - T(x',y'), g(x) - g(x') \rangle \geq \nu \left\| g(x) - g(x') \right\|^2$$

holds, for all $y, y' \in H$;

(ii) (g, ω) cocoercive, if for each $x, x' \in H$, there exists a constant $\omega > 0$ such that

$$\langle T(x,y) - T(x',y'), g(x) - g(x') \rangle \geq -\omega \left\| T(x,y) - T(x',y') \right\|^2$$

holds, for all $y, y' \in H$;

(iii) relaxed (g, ω, t) cocoercive, if for each $x, x' \in H$, there exists constants $t > 0$ and $\omega > 0$ such that

$$\begin{aligned}\langle T(x,y) - T(x',y'), g(x) - g(x') \rangle \geq -\omega \left\| T(x,y) - T(x',y') \right\|^2 \\ + t \left\| g(x) - g(x') \right\|^2\end{aligned}$$

holds, for all $y, y' \in H$;

(iv) (g, μ) Lipschitz continuous in the first variable, if for each $x, x' \in H$, there exists a constant $\mu > 0$ such that

$$\left\| T(x,y) - T(x',y') \right\| \leq \mu \left\| g(x) - g(x') \right\|$$

holds, for all $y, y' \in H$.

Definition 6. A mapping $g : H \to H$ is said to be $\zeta-$expansive if for all $x, y \in H$, there exists a constant $\zeta > 0$, such that

$$\|g(x) - g(y)\| \geq \zeta \|x - y\| \ .$$

Lemma 7. (see [10]) *Let $\{a_n\}$ and $\{b_n\}$ be two nonnegative real sequences satisfying the following conditions:*

$$a_{n+1} \leq (1 - d_n)a_n + b_n \, , \quad \forall \, n \geq n_0 \, ,$$

where n_0 is some nonnegative integer, $d_n \in (0,1)$ with $\sum_{n=0}^{\infty} d_n = \infty$ and $b_n = o(d_n)$, then $a_n \to 0$ (as $n \to \infty$).

Theorem 8. *Let $g : H \to H$ be $\zeta-$expansive and $T_i : H \times H \to H$ be two-variable relaxed (g, ω_i, t_i)-cocoercive and (g, μ_i)-Lipschitz mapping in the first variable for $i = 1, 2$. In addition, the following assumptions hold:*

(i) $0 \le \rho \le \frac{2\left(t_1 - \omega_1 \mu_1^2\right)}{\mu_1^2}$; $t_1 > \omega_1 \mu_1^2$,

(ii) $0 \le \eta \le \frac{2\left(t_2 - \omega_2 \mu_2^2\right)}{\mu_2^2}$; $t_2 > \omega_2 \mu_2^2$,

(iii) $0 \le \alpha_n, \beta_n \le 1$, $\sum_{n=0}^{\infty} \alpha_n = \infty$, $\lim_{n\to\infty}(1 - \beta_n) = 0$.

Then the sequences $\{x_n\}$ and $\{y_n\}$ generated by (8) converges to x^* and y^* respectively.

Proof. For $x^*, y^* \in H$, by Lemma 4, we have

$$\begin{aligned} g(x^*) &= J_\varphi \left[g(y^*) - \rho T_1(y^*, x^*)\right], \\ g(y^*) &= J_\varphi \left[g(x^*) - \eta T_2(x^*, y^*)\right], \end{aligned} \tag{12}$$

To prove the result, we first evaluate $\|g(x_{n+1}) - g(x^*)\|$ for all $n \ge 0$. Using (8), we obtain

$$\begin{aligned} &\|g(x_{n+1}) - g(x^*)\| \\ &\quad \le \|(1 - \alpha_n) g(x_n) + \alpha_n J_\varphi \left[g(y_n) - \rho T_1(y_n, x_n)\right] - g(x^*)\| \\ &\quad \le (1 - \alpha_n) \|g(x_n) - g(x^*)\| \\ &\qquad + \alpha_n \|J_\varphi \left[g(y_n) - \rho T_1(y_n, x_n)\right] - J_\varphi \left[g(y^*) - \rho T_1(y^*, x^*)\right]\| \\ &\quad \le (1 - \alpha_n) \|g(x_n) - g(x^*)\| \\ &\qquad + \alpha_n \|g(y_n) - g(y^*) - \rho \left[T_1(y_n, x_n) - T_1(y^*, x^*)\right]\| . \end{aligned} \tag{13}$$

Since T_1 is relaxed (g, ω_1, t_1)-cocoercive and (g, μ_1)-Lipschitz mapping in the first variable, we have

$$\begin{aligned} &\|g(y_n) - g(y^*) - \rho \left(T_1(y_n, x_n) - T_1(y^*, x^*)\right)\|^2 \\ &\quad = \|g(y_n) - g(y^*)\|^2 - 2\rho \left\langle T_1(y_n, x_n) - T_1(y^*, x^*), g(y_n) - g(y^*)\right\rangle \\ &\qquad + \rho^2 \|T_1(y_n, x_n) - T_1(y^*, x^*)\|^2 \\ &\quad \le \|g(y_n) - g(y^*)\|^2 + 2\rho\omega_1 \|T_1(y_n, x_n) - T_1(y^*, x^*)\|^2 \\ &\qquad - 2\rho t_1 \|g(y_n) - g(y^*)\|^2 + \rho^2 \|T_1(y_n, x_n) - T_1(y^*, x^*)\|^2 \\ &\quad \le \|g(y_n) - g(y^*)\|^2 + 2\rho\omega_1\mu_1^2 \|g(y_n) - g(y^*)\|^2 \\ &\qquad - 2\rho t_1 \|g(y_n) - g(y^*)\|^2 + \rho^2\mu_1^2 \|g(y_n) - g(y^*)\|^2 \\ &\quad = \left[1 + 2\rho\omega_1\mu_1^2 - 2\rho t_1 + \rho^2\mu_1^2\right] \|g(y_n) - g(y^*)\|^2 \end{aligned} \tag{14}$$

By (13) and (14), we have

$$\begin{aligned} \|g(x_{n+1}) - g(x^*)\| \leq (1-\alpha_n)\|g(x_n) - g(x^*)\| \\ + \alpha_n\theta_1 \|g(y_n) - g(y^*)\| \,, \end{aligned} \tag{15}$$

where $\theta_1 = \sqrt{1 + 2\rho\omega_1\mu_1^2 - 2\rho t_1 + \rho^2\mu_1^2}$. Again, from (8), we have

$$\begin{aligned} \|g(y_n) - g(y^*)\| &\leq \|(1-\beta_n)g(x_n) + \beta_n J_\varphi\left[g(x_n) - \eta T_2(x_n, y_n)\right] - g(y^*)\| \\ &\leq (1-\beta_n)\|g(x_n) - g(y^*)\| \\ &\quad + \beta_n \|g(x_n) - g(x^*) - \eta\left[T_2(x_n, y_n) - T_1(x^*, y^*)\right]\| \,. \end{aligned} \tag{16}$$

Since T_2 is relaxed (g, ω_2, t_2)-cocoercive and (g, μ_2)-Lipschitz mapping in the first variable, we have

$$\begin{aligned} \|g(x_n) - g(x^*) - \eta\left[T_2(x_n, y_n) - T_1(x^*, y^*)\right]\| \\ \leq \theta_2 \|g(x_n) - g(x^*)\| \end{aligned} \tag{17}$$

where $\theta_2 = \sqrt{1 + 2\eta\omega_2\mu_2^2 - 2\eta t_2 + \eta^2\mu_2^2}$.

Substituting (17) into (16), we get

$$\begin{aligned} \|g(y_n) - g(y^*)\| &\leq (1-\beta_n)\|g(x_n) - g(y^*)\| + \beta_n\theta_2\|g(x_n) - g(x^*)\| \\ &\leq (1-\beta_n)\left(\|g(x_n) - g(x^*)\| + \|g(x^*) - g(y^*)\|\right) \\ &\quad + \beta_n\theta_2\|g(x_n) - g(x^*)\| \\ &= \left[1 - \beta_n(1-\theta_2)\right]\|g(x_n) - g(x^*)\| \\ &\quad + (1-\beta_n)\|g(x^*) - g(y^*)\| \,. \end{aligned} \tag{18}$$

Substituting (18) into (15), we get

$$\begin{aligned} \|g(x_{n+1}) - g(x^*)\| &\leq (1-\alpha_n)\|g(x_n) - g(x^*)\| \\ &\quad + \alpha_n\theta_1\left[1 - \beta_n(1-\theta_2)\right]\|g(x_n) - g(x^*)\| \\ &\quad + \alpha_n\theta_1(1-\beta_n)\|g(x^*) - g(y^*)\| \\ &= \left[1 - \alpha_n\left\{1 - \theta_1(1 - \beta_n\left(1-\theta_2\right))\right\}\right]\|g(x_n) - g(x^*)\| \\ &\quad + \alpha_n\theta_1(1-\beta_n)\|g(x^*) - g(y^*)\| \end{aligned} \tag{19}$$

Set

$$\begin{aligned} a_n &= \|g(x_{n+1} - g(x_n)\| \,, \\ b_n &= \alpha_n\theta_1(1-\beta_n)\|g(x^*) - g(y^*)\| \,, \end{aligned}$$

and

$$d_n = \alpha_n \left\{1 - \theta_1(1 - \beta_n (1 - \theta_2))\right\}.$$

Since $\theta_1, \theta_2 < 1$, we have $d_n \in (0,1)$ for all $n \geq 0$. Condition (i) and (ii) implies that $b_n = o(d_n)$. Also by condition (iii), we can see that $d_n > \alpha_n(1-\theta_1)$ for all $n \geq 0$ and so from the condition $\sum_{n=0}^{\infty} \alpha_n < \infty$, we obtain $\sum_{n=0}^{\infty} d_n < \infty$. Hence by Lemma 7, we have

$$\lim_{n\to\infty} \|g(x_n) - g(x^*)\| = 0\,, \quad \text{and} \quad \lim_{n\to\infty} \|g(y_n) - g(y^*)\| = 0\,.$$

Since g is expansive, we have

$$\lim_{n\to\infty} x_n = x^*, \quad \text{and} \quad \lim_{n\to\infty} y_n = y^*\,.$$

This completes the proof. □

Remark 1. Theorem 8 extends and generalizes the main result in [3], which itself is an extension and improvement of the main result in Chang et al. [2].

As a consequence of Theorem 8, we have the following results:

Corollary 9. *Let $g : H \to H$ be $\zeta-$expansive and $T_i : H \to H$ be univariate relaxed (g, ω_i, t_i)-cocoercive and (g, μ_i)-Lipschitz mapping for $i = 1, 2$. In addition, the following assumptions hold:*

(i) $0 \leq \rho \leq \frac{2\left(t_1 - \omega_1\mu_1^2\right)}{\mu_1^2}$; $t_1 > \omega_1\mu_1^2$,

(ii) $0 \leq \eta \leq \frac{2\left(t_2 - \omega_2\mu_2^2\right)}{\mu_2^2}$; $t_2 > \omega_2\mu_2^2$,

(iii) $0 \leq \alpha_n, \beta_n \leq 1$, $\sum_{n=0}^{\infty} \alpha_n = \infty$, $\lim_{n\to\infty}(1-\beta_n) = 0$.

Then the sequences $\{x_n\}$ and $\{y_n\}$ generated by (9) converge to x^* and y^*, respectively.

Corollary 10. *Let $g : H \to H$ be $\zeta-$expansive and $T_i : H \times H \to H$ be two-variable relaxed (g, ω_i, t_i)-cocoercive and (g, μ_i)-Lipschitz mapping in the first variable for $i = 1, 2$. In addition, the following assumptions hold:*

(i) $0 \leq \rho \leq \frac{2\left(t_1 - \omega_1\mu_1^2\right)}{\mu_1^2}$; $t_1 > \omega_1\mu_1^2$,

(ii) $0 \leq \eta \leq \frac{2(t_2 - \omega_2 \mu_2^2)}{\mu_2^2}$; $t_2 > \omega_2 \mu_2^2$,

(iii) $0 \leq \alpha_n \leq 1$, $\sum_{n=0}^{\infty} \alpha_n = \infty$.

Then the sequences $\{x_n\}$ *and* $\{y_n\}$ *generated by* (10) *converge to* x^* *and* y^**, respectively.*

Corollary 11. *Let* $g : H \to H$ *be* $\zeta-$*expansive and* $T_i : H \to H$ *be univariate relaxed* (g, ω_i, t_i)*-cocoercive and* (g, μ_i)*-Lipschitz mapping for* $i = 1, 2$. *In addition, the following assumptions hold:*

(i) $0 \leq \rho \leq \frac{2(t_1 - \omega_1 \mu_1^2)}{\mu_1^2}$; $t_1 > \omega_1 \mu_1^2$,

(ii) $0 \leq \eta \leq \frac{2(t_2 - \omega_2 \mu_2^2)}{\mu_2^2}$; $t_2 > \omega_2 \mu_2^2$,

(iii) $0 \leq \alpha_n \leq 1$, $\sum_{n=0}^{\infty} \alpha_n = \infty$.

Then the sequences $\{x_n\}$ *and* $\{y_n\}$ *generated by* (11) *converge to* x^* *and* y^**, respectively.*

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